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An Exposition of Constructivism: Why Some Like it Radical

Man, having within himself an imagined World of lines and numbers, operates in it with abstractions, just as God, in the universe, did with reality—Giambattista Vico¹

When the Neapolitan philosopher Giambattista Vico published his treatise on the construction of knowledge,² it triggered quite a controversy in the Giornale de'Letterati d'Italia, one of the most prestigious scholarly journals at the time. This was in the years 1710–12. The first reviewer, who remained anonymous, had carefully read the treatise and was obviously shocked by the implications it had for traditional epistemology—all the more so because, as he conceded, the arguments showed great learning and were presented with elegance. He was therefore impelled to question Vico's position, and he very politely suggested that one thing was lacking in the treatise: the proof that what it asserted was true.³

Today, those constructivists who are "radical" because they take their theory of knowing seriously, frequently meet the same objection—except that it is sometimes expressed less politely than at the beginning of the 18th century. Now, no less than then, it is difficult to show the critics that what they demand is the very thing constructivism must do without. To claim that one's theory of knowing is true, in the traditional sense of representing a state or feature of an experiencer-independent world, would be perjury for a radical constructivist. One of the central points of the theory is precisely that this kind of "truth," can never be claimed for the knowledge (or any piece of it) that human reason produces. To mark this radical departure, I have in the last few years taken to calling my orientation a theory of knowing rather than a "theory of knowledge." I agree whole-heartedly with Noddings when she says, at the beginning of her contribution to this volume, that radical constructivism should be "offered as a post-epistemological perspective." One of the consequences of such an appraisal, however, must be that one does not persist in arguing against it as though it were or purported to be a traditional theory of knowledge. Another consequence-for me the more important one—is that constructivism needs to be radical and must explain that one can, indeed, manage without the traditional notion of Truth. That this task is possible, may become more plausible if I trace the sources of some of the ideas that made the enterprise seem desirable.

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In retrospect, the path along which I picked up relevant ideas (somewhat abbreviated and idealized) led from the early doubts of the Pre-Socratics, via Montaigne, Berkeley, Vico, and Kant, to thinkers who developed instrumentalism and pragmatism at the turn of this century, and eventually to the Italian Operational School and Piaget's genetic epistemology.

The Way of the Sceptics

To Xenophanes (6th century B.C.) we may credit the insight that even if someone succeeded in describing exactly how the world really is, he or she would have no way of knowing that it was the "true" description.⁴ This is the major argument the sceptics have repeated for two thousand five hundred years. It is based on the assumption that whatever ideas or knowledge we have must have been derived in some way from our experience, which includes sensing, acting, and thinking. If this is the case, we have no way of checking the truth of our knowledge with the world presumed to be lying beyond our experiential interface, because to do this, we would need an access to such a world that does not involve our experiencing it.

Plato tried to get around this by claiming that some god had placed the pure ideas inside us and that experience with the fuzzy, imperfect world of the senses could only serve to make us "remember" what was really true. Thus, there would be no need (and no way) to check our knowledge against an independent external reality. Consequently, in Plato's famous metaphor, the man who is led out of the cave of his commonplace experience is blinded by a splendid vision. But his vision is the pure realm of an interpersonal soul and not the fuzzy world perceived by the senses.⁵ From my point of view, Plato created an ingenious poetic or "metaphysical" myth, but not a rational theory of knowing.

The sceptics position, developed into a school under Pyrrho at the end of the next century, was diligently compiled and documented by Sextus Empiricus about 200 A.D. It smoldered under the theological debates of the middle ages and burst into full flame in the 16th century when the works of Sextus Empiricus were rediscovered. Descartes set out to put an end to it, but succeeded only in strengthening the side he was opposing (cf. Popkin, 1979). The British Empiricists then helped to harden the sceptical doctrine by their detailed analyses. First, Locke discarded the secondary (sensory) properties of things as sources of "true" information about the real world. Then, Berkeley showed that Locke's arguments applied equally to the primary properties (spatial extension, motion, number, etc.), and finally Hume delivered an even more serious blow by attributing the notion of causality (and other relations that serve to organize experience) to the conceptual habits of the human knower. The final demolition of realism was brought about when Kant suggested that the concepts of space and time were the necessary forms of human experience, rather than characteristics of the universe. This meant that we cannot even imagine what the structure of the real world might be like, because whatever we call structure is necessarily an arrangement in space, time, or both.

These are extremely uncomfortable arguments. Philosophers have forever tried to dismantle them, but they have had little success. The arguments are uncomfortable because they threaten a concept which we feel we cannot do without. "Knowledge" is something of which we are quite sure that we have a certain amount, and we are not

prepared to relinquish it. The trouble is that throughout the occidental history of ideas and right down to our own days, two requisites have been considered fundamental in any epistemological discussion of knowledge. The first of these requisites demands that whatever we would like to call "true knowledge" has to be independent of the knowing subject. The second requisite is that knowledge is to be taken seriously only if it claims to represent a world of "things-in-themselves" in a more or less veridical fashion. In other words, it is tacitly taken for granted that a fully structured and knowable world "exists" and that it is the business of the cognizing human subject to discover what that structure is.

The weakness of the sceptics' position lies in its polemical formulation. It always sounds as though the traditional epistemologists' definition of knowledge were the only possible one. Hence, when Montaigne says "*la peste de l'homme c'est l'opinion de savoir*" (mankind's plague is the conceit of knowing),⁶ it sounds as though we ought to give up all knowing. But he was referring to absolutistic claims of experiential knowledge and was discussing them in the context of the traditional dogmatic belief that religious revelation is unquestionable. He had in mind absolute truth, and he was castigating those who claimed that a rational interpretation of experience (of which "scientific observation" is, after all, a sophisticated form) would lead to such truth. He certainly did not intend to discredit the kind of know-how that enabled his peasants to make a good wine.

In short, what the sceptics failed to stress was that, though no truths about a "real" world could be derived from experience, experience nevertheless supplied a great deal of useful knowledge.

The Changed Concept of Knowledge

Unbeknownst to Kant, who in the 1780s hammered this limitation in with his Critiques of pure and practical reason, Giambattista Vico had come to a very similar conclusion in 1710. The human mind can know only what the human mind has made, was his slogan and, more like Piaget than Kant, he did not assume that space and time were necessarily a priori categories, but suggested that they, too, were human constructs (Vico, 1858).

Pursuing this way of thinking, one is led to what I have called "a reconstruction of the concept of knowledge" (von Glasersfeld, 1985). Some reconstruction is needed because, on the one hand, one can no longer maintain that the cognizing activity should or could produce a true representation of an objective world, and on the other, one does not want to end up with a solipsistic form of idealism. The only way out, then, would seem to be a drastic modification of the relation between the cognitive structures we build up and that "real" world which we are inclined to assume as "existing" beyond our perceptual interface.⁷ Instead of the illusory relation of "representation," one has to find a way of relating knowledge to reality that does not imply anything like match or correspondence.

Neither Vico nor Kant explicitly mentioned such a conceptual alternative. It was supplied, however, in Darwin's theory of evolution by the concept of fit. Once this relational concept has been stripped of its erroneous formulation in the slogan "survival of the fittest" (cf. Pittendrigh, 1958; von Glasersfeld, 1980), it offers a way around the paradox of the traditional theory of knowledge. As far as I know, this was first suggested by Willam James (1880).⁸ Georg Simmel (1885) elaborated it, and Aleksandr Bogdanov (1909) developed it into a comprehensive instrumentalist epistemology. Hans Vaihinger (1913), who had been working at his "Philosophy of As If" since the 1870s and who probably was quite unaware of Vico, reintroduced the idea of conceptual construction.

Piaget's Contribution

Today, in retrospect, these and other authors can be cited as "sources" of constructivism. However, the great pioneer of the constructivist theory of knowing today, Jean Piaget started from Kant and arrived at his view of cognition as a biologist who looked at intelligence and knowledge as biological functions whose development had to be explained and mapped in the ontogeny of organisms.

In interpreting Piaget, it is important to remember that his publications range over an astounding variety of topics and are spread over more than half a century.⁹ As with any versatile and original thinker, his ideas did not cease to develop and change (Vuik, 1981). It is, therefore, not surprising that one can spot contradictions in his work. An obvious instance is his theory of stages, which was gradually superseded by his theory of equilibration (cf. Rowell, in press). Thus it is not too difficult to dismiss Piaget on the strength of one or two quotations; or, what is even more frequent, on the strength of what superficial summarizers have said about him. It is also likely that arguments about what Piaget actually believed will continue and that different scholars will provide different interpretations. In my view, the following basic principles of radical constructivism emerge quite clearly if one tries to comprise as much as possible of Piaget's writings in one coherent theory—but I would argue for these principles even if they could be shown to diverge from Piaget's thinking.

(1a) Knowledge is not passively received either through the senses or by way of communication;

(1b) knowledge is actively built up by the cognizing subject.

(2a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;

(2b) cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality.

One cannot adopt these principles casually. If taken seriously, they are incompatible with the traditional notions of knowledge, truth, and objectivity, and they require a radical reconstruction of one's concept of reality. Instead of an inaccessible realm beyond perception and cognition, it now becomes the experiential world we actually live in. This world is not an unchanging independent structure, but the result of distinctions that generate a physical and a social environment to which, in turn, we adapt as best we can.

Consequently, one cannot adopt the constructivist principles as an absolute truth, but only as a working hypothesis that may or may not turn out to be viable. This is the main reason why the constructivist orientation is unequivocally post-epistemological (Noddings, this volume).

The Concept of Viability

To relinquish the inveterate belief that knowledge must eventually represent something that lies beyond our experience is, indeed, a frightening step to take. It constitutes a feat of decentering that is even more demanding than the one accomplished by a few outstanding thinkers in the 16th century who realized that the earth was not the center of the universe. Because it goes against an age-old habit, it is immensely difficult to accept that, no matter how well we can predict the results of certain actions we take or the "effects" of certain "causes" we observe, this must never be interpreted as a proof that we have discovered how the "real" world works.¹⁰

The key to this insight lies in what Piaget formulated in the phrase "*l'objet se laisse faire*" ("the object allows itself to be treated"; 1970; p.35) At the symposium on the occasion of his 80th birthday he repeated the phrase and explained it further: "When one comes to have a true theory, this is because the object permitted it; which amounts to saying that it contained something analogous to my actions." (Inhelder et al. 1977; p.64) In this context—as in so many in Piaget's works—it is important to remember that an "object" is never a thing-in-itself for Piaget, but something that the cognizing subject has constructed by making distinctions and coordinations in his or her perceptual field (Piaget, 1937).

That is all very well, one might say, but how does it come about that the reality we construct is in many ways remarkably stable? And, one might also ask why, if we ourselves construct our experiential reality, can we not construct any reality we might like? The first question was answered in a categorical way by George Kelly: "To the living creature, then, the universe is real, but it is not inexorable unless he chooses to construe it that way" (1955; p.8). The living creature, be it fish, fowl, or human, thrives by abstracting regularities and rules from experience that enable it to avoid disagreeable situations and, to some extent, to generate agreeable ones. This "abstracting of regularities" is always the result of assimilation. No experience is ever the same as another in the absolute sense. Repetition and, consequently, regularity can be obtained only by disregarding certain differences. This notion of assimilation is at the core of Piaget's scheme theory. No schemes could be developed if the organism could not isolate situations in which a certain action leads to a desirable result. It is the focus on the result that distinguishes a scheme from a reflex and makes possible the form of learning that Piaget called accommodation. It takes place when a scheme does not lead to the expected result. This produces a perturbation, and the perturbation may lead either to a modification of the pattern that was abstracted as the "triggering situation" or to a modification of the action. All this, I want to emphasize, concerns the experiential world of the acting organism, not any "external" reality. And the patterns a cognizing organism can and does abstract from experience depend on the operations of distinction and coordination the organism can and does carry out.¹¹ This was brilliantly demonstrated for a variety of organisms more than fifty years ago by Jakob von Uexküll (1933/1970).

The second question—why we cannot construct any reality we like—can be raised only if the concept of viability is misunderstood or ignored. The absurdity of solipsism stems from the denial of any relation between knowledge and an experiencerindependent world. Radical Constructivism has been careful to stress that all action, be it physical or conceptual, is subject to constraints. I can no more walk through the desk in front of me than I can argue that black is white at one and the same time. What constrains me, however. is not quite the same in the two cases. That the desk constitutes an obstacle to my physical movement is due to the particular distinctions my sensory system enables me to make and to the particular way in which I have come to coordinate them. Indeed, if I now could walk through the desk, it would no longer fit the abstraction I have made in prior experience. This, I think, is simple enough. What is not so simple is the realization that the fact that I am able to make the particular distinctions and coordinations and establish their permanence in my experiential world, does not tell me anything other than the fact that it is one of the things my experiential reality allows me to do. Using a spatial metaphor, I have at times expressed this by saying that the viability of an action shows no more than that the "real" world leaves us room to act in that way. Conversely, when my actions fail and I am compelled to make a physical or conceptual accommodation, this does not warrant the assumption that my failure reveals something that "exists" beyond my experience. Whatever obstacle I might conjecture, can be described only in terms of my own actions. (In this context, it is important to remember that the constructivist theory holds that perception is not passive, but under all circumstances the result of action; cf. Piaget, 1969.)

The constraints that preclude my saying that black is white are, of course, not physical but conceptual. The way we use symbols to handle abstractions we have made from experience, requires among other things that we exclude contradiction (cf. von Glasersfeld, in press). Consistency, in maintaining semantic links and in avoiding contradictions, is an indispensable condition of what I would call our "rational game."

The Question of Certainty

The domain of mathematics is in some sense the epitome of the rational game. The certainty of mathematical results has often been brought up as an argument against constructivism.

To indicate that the theoretical infallibility of mathematical operations (in practice, mistakes may, of course, occur) cannot be claimed as proof that these operations give access to an ontological reality, I have compared this generation of certainty to the game of chess. At the painful moment when you discover that your opponent can put you into a "checkmate" position, you have no way of doubting it and your shock is as real as any shock can be. Yet, it is obvious that the certainty you are experiencing springs from nothing but the conceptual relations that constitute the rules of the game; and it is equally obvious that these conceptual relations are absolute in the sense that if I broke them and thus destroyed the certainty they generate, I would no longer be playing that particular game.

The comparison with chess has caused remonstrations, and I would like to clarify my position. I still believe that the certainty in mathematics springs from the same conceptual source, but this does not mean that I hold mathematics to be like chess in other ways. The biggest difference is that the elements to which the rules of chess apply are all specific to the game. Flesh and blood kings cannot be put into "mate" positions, equestrian knights move unlike their chess namesakes, and living queens show their power in ways that are inconceivable on the chess board. In contrast, the elements to which the rules of mathematics are applied, are not free inventions. In counting, for example, the elements start out as ordinary things that have been abstracted from ordinary experience, and the basic abstract concepts, such as "oneness" and "plurality," have a life of their own before they are incorporated in the realm of mathematics. It is precisely this connection with everyday experience and conceptual practice that leads to the contention that mathematics "reflects" the real world.

The "imagined world of lines and numbers" of which Vico speaks in the quotation I have put at the beginning of this essay, is in no sense an arbitrary world. At the roots of the vast network of mathematical abstractions are the simple operations that allow us to perceive discrete items in the field of our experience, and simple relational concepts that allow us to unite them as "units of units." On subsequent levels of abstraction, the re-presentations of sensory-motor material of everyday experience (Piaget's "figurative" elements) drop out, and what remains is the purely "operative," i.e., abstractions from operations.

None of this is developed in a free, wholly arbitrary fashion. Every individual's abstraction of experiential items is constrained (and thus guided) by social interaction and the need of collaboration and communication with other members of the group in which he or she grows up. No individual can afford not to establish a relative fit with the consensual domain of the social environment.¹²

An analogous development takes place with regard to mathematics, but here the social interaction specifically involves those who are active in that field. The consensual domain into which the individual must learn to fit is that of mathematicians, teachers, and other adults insofar as they practice mathematics. The process of adaptation is the same as in other social domains, but there is an important difference in the way the degree of adaptation can be assessed. In the domain of everyday living, fit can be demonstrated by sensory-motor evidence of successful interaction (e.g. when an individual asked to buy apples, returns with items that the other recognizes as apples). The only observable manifestation of the demand as well as of the response, in the abstract reaches of the domain of mathematics, are symbols of operations. The operations themselves remain unobservable. Understanding can therefore never be demonstrated by the presentation of results that may have been acquired by rote learning.¹³ This is one of the reasons why mathematics teachers often insist (to the immense boredom of the students) on the exact documentation of the algorithm by means of which the result was obtained. The flaw in this procedure is that any documentation of an algorithm is again a sequence of symbols which in themselves do not demonstrate the speaker's or writer's understanding of the symbolized operations. Hence, the production of such a sequence, too, may be the result of rote learning.

Other contributions to this volume will illustrate how a constructivist approach to instruction deals with this problem. They will also show that the constructivist teacher does not give up his or her role as a guide—but this leadership takes the form of encouraging and orienting the students' constructive effort rather than curtailing their autonomy by presenting ready-made results as the only permitted path.

Here, I would merely stress the sharp distinction which, in my view, has to be made between teaching and training. The first aims at the students' conceptual fit with the consensual domain of the particular field, a fit which, from the teacher's perspective, constitutes understanding. The second aims at the students' behavioral fit which, from the teacher's perspective, constitutes acceptable performance. This is not to say, that rote learning and the focus on adequate performance should have no place in constructively oriented instruction. But it does mean that, where the domain of mathematics is concerned, instruction that focuses on performance alone can be no better than trivial.

Concluding Remarks

If one seriously wants to adopt the radical constructivist orientation, the changes of thinking and of attitudes one has to make are formidable. It is also far from easy to maintain them consequentially. Much like physical habits, old ways of thinking are slow to die out and tend to return surreptitiously.

In everyday living we don't risk much if we continue to speak of lovely sunsets and say that tomorrow the sun will rise at such and such a time—even though we now hold that it is the earth that moves and not the sun. Similarly, there is no harm in speaking of knowledge, mathematical and other, as though it had ontological status and could be "objective" in that sense; as a way of speaking this is virtually inevitable in the social interactions of everyday life. But when we let scientific knowledge turn into belief and begin to think of it as unquestionable dogma, we are on a dangerous slope.

The critics of Copernicus who argued that his system must be "wrong" because it denied that the earth is the center of the universe, could not claim to be "scientific"— they argued in that way for political and religious reasons. Science, as Bellarmino pointed out, produces hypotheses, and as such, they may or may not be useful. Their use may also be temporary. The science we have today, holds that neither the earth nor the sun has a privileged position in the universe. Like the contemporary philosophers of science, constructivists have tried to learn from that development and to give up the tradiional conception of knowledge as a "true" representation of an experiencer-independent state of affairs. That is why radical constructivism does not claim to have found an ontological truth but merely proposes a hypothetical model that may turn out to be a useful one.

Let me conclude with a remark that is not particularly relevant to the teaching of mathematics but might be considered by educators in general. Throughout the two thousand five hundred years of Western epistemology, the accepted view has been a realist view. According to it, the human knower can attain some knowledge of a really existing world and can use this knowledge to modify it. People tended to think of the world as governed by a God who would not let it go under. Then faith shifted from God to science and the world that science was mapping was called "Nature" and believed to be ultimately understandable and controllable. Yet, it was also believed to be so immense that mankind could do no significant harm to it. Today, one does not have to look far to see that this attitude has endangered the world we are actually experiencing.

If the view is adopted that "knowledge" is the conceptual means to make sense of experience, rather than a "representation" of something that is supposed to lie beyond it, this shift of perspective brings with it an important corollary: the concepts and relations in terms of which we perceive and conceive the experiential world we live in

are necessarily generated by ourselves. In this sense it is we who are responsible for the world we are experiencing. As I have reiterated many times, radical constructivism does not suggest that we can construct anything we like, but it does claim that within the constraints that limit our construction there is room for an infinity of alternatives. It therefore does not seem untimely to suggest a theory of knowing that draws attention to the knower's responsibility for what the knower constructs.

Footnotes

- 1. Vico's reply to his critics, included in the 2nd edition of De Antiquissima Italorum Sapientia, 1711; reprinted in Vico (1858) p.143.
- 2. De Antiquissima Italorum Sapientia, Naples, 1710; reprinted with Italian translation, 1858.
- 3. Giornale de'Letterati d'Italia, 1711, vol.V, article VI; reprinted in Vico (1858), p. 137.
- 4. cf. Hermann Diels (1957), Xenophanes, fragment 34.
- 5. cf. Plato's "The Republic" in Great Dialogues of Plato (1956), p. 312ff.
- 6. Montaigne wrote this in his Apologie de Raymond Sebond (1575–76); cf. Essais, 1972, vol.2,, p.139.
- 7. Though most philosophers, today, would agree that the ontological realm is perceptually inaccessible, they balk at Kant's suggestion that it is also conceptually inaccessible to us. They are therefore still stuck with the paradox that they have no way of showing the truth of the ontological claims they make.
- 8. This reference was brought to my attention by a personal communication from Jacques Vonèche (Geneva, 1985).
- 9. See, for instance, Kitchener's recent article (1989) on Piaget's early work on the role of social interaction and exchange.
- 10. Paul Feyerabend's recent comment (1987) on the famous letter Cardinal Bellarmino wrote in the context of Galileo's trial, makes this point in exemplary fashion: "To use modern terms: astronomers are entirely safe when saying that a model has predictive advantages over another model, but they get into trouble when asserting that it is therefore a faithful image of reality. Or, more generally: the fact that a model works does not by itself show that reality is structured like the model." (p.250)
- 11. The focus on "operations of distinction" is a major contribution of Humberto Maturana's biological approach to cognition (1980); the notion as such, however, is implicit in much of Piaget's work, e.g., his Mechanisms of perception (1969).
- 12. Lest this be interpreted as a concession to realism, let me point out that, in the constructivist view, the term "environment" always refers to the environment as experientially constructed by the particular subject, not to an "objective" external world.
- 13. Thinking, conceptual development, understanding, and meaning are located in someone's head and are never directly observable. A formidable confusion was generated by the behaviorist program that tried to equate meaning with observable response.

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